Collaborative Research: Initiating a Foundational Research Model for Secondary Mathematical Knowledge for Teaching (INFORMS MKT)

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1. Goals and Objectives

We propose foundational research situated in teacher practice to contribute to the understanding of the nature and quality of secondary mathematics teachers' decisions involving their mathematical knowledge for teaching (MKT). As researchers and educators in mathematics teacher education, we share the generally accepted view that teachers' MKT is strongly related to their effectiveness and that more research is needed to advance the field's understanding of how to conceptualize, develop, and assess teachers' MKT (National Resource Council, 2012). Our goal is to understand the nature and role of MKT in expert practice in order to contribute to the development of models of effective MKT and ultimately to inform and improve teacher development. This proposed research addresses the call in the EHR Core Research Program to engage in research on STEM learning environments, such as the interface between teaching and learning. In particular, we focus on how teachers' use of MKT in their work of teaching mediates the quality of students' opportunity for mathematical reasoning.

Our research is grounded in a model of the relationships among teachers' MKT, the work of teaching, and students' mathematical learning (Figure 1). In this model, the work of teaching is decomposed into three interacting areas: planning, instruction, and assessment, where assessment includes both teachers' assessment of student learning and teachers' reflection on instruction. A teacher's MKT influences how these areas are enacted via the use of this knowledge in teachers' decision making. Student learning is influenced by the opportunities for students to engage in mathematical reasoning created by the enactment of planning, instruction, and assessment. Each relationship in our model is bi-directional and heavily mediated by contextual factors. For example, a teacher may have MKT related to a particular topic, but not draw on this knowledge in their planning because the topic is not included in the existing curriculum. With respect to student learning, how a student chooses to engage with an opportunity to reason is a factor that influences their learning.

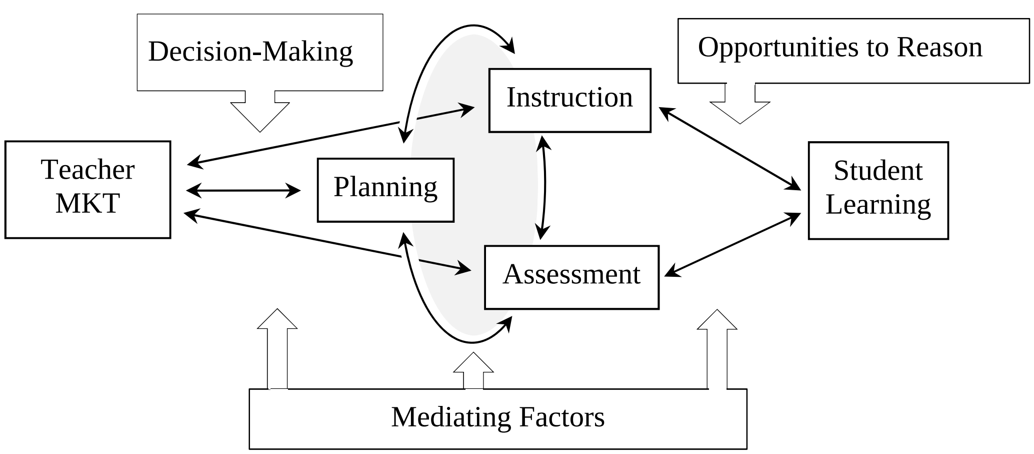

Figure 1. Relationships among teacher knowledge, the work of teaching, and student learning.

No single definition exists for the mathematical knowledge teachers need to effectively teach mathematics, although it is generally accepted that this knowledge consists of knowledge that is purely mathematical and knowledge that is related to the pedagogy of teaching mathematics effectively. For the purposes of our research, we use the term mathematical knowledge for teaching (MKT) to encompass both types of knowledge. Further, we use the idea that MKT is the mathematical knowledge, skills, and sensibilities entailed in the work of teaching mathematics to students (Ball, Thames, & Phelps, 2008). We operationalize MKT as the ideas teachers can or do use in the work of teaching that have a mathematical component. Thus, we identify MKT by its mathematical nature and its employment or potential employment in the work of teaching.

Our research questions are:

1. (Enactment) What are the decisions expert teachers make while engaged in the work of teaching exponential functions that involve MKT?
2. (Nature) What is the MKT these teachers draw on while making these decisions?
3. (Mediation) What factors mediate the use of teachers' MKT in their decision making?
4. (Quality) What is the quality of these teachers' decisions with respect to students' opportunity to reason mathematically and how is this quality related to the types of decisions they make, the nature of their MKT, the factors that mediate the decisions?

The research questions and methods for our research are designed to address the following specific needs for MKT research.

Need for research on MKT in teachers' practice. Because MKT is embedded in the work of teaching, there is a need for research focused on teachers' practice (Ball, 2011; Ball, Lubienski, & Mewborn, 2001; Barwell, 2013; Speer & King, 2009; Thompson, 2013). Kajander (2010) suggests that current descriptions of MKT categories provide a general view of MKT, but lack the depth and specificity needed by teachers to change various aspects of their practice. A reason for this is that much current MKT research treats MKT as declarative knowledge rather than one's knowledge to act (Thompson, in press). Similarly, Hashweh (2005) observes, "that in our efforts to understand teacher knowledge and thinking we have focused on knowledge at the expense of thinking processes," which includes how teachers integrate and act on the various aspects of MKT knowledge. Drawing on decision making as the core work of teaching (Shavelson, 1973), we use the lens of decision making to examine MKT in practice.

Need to account for factors mediating teachers' application of their MKT. Kajander (2010) argues that "the teachers' context and viewpoint should remain an important voice in furthering understanding of the domain of mathematics for teaching" (p. 88). Teachers' decisions are strongly influenced by contextual factors such as curriculum resources, school demographics, school leadership, and school policy (Campbell et al., 2014; Petrou & Goulding, 2011). Hill and Charalambous (2012) suspect that contextual factors, such as teachers' perceptions of the need to 'cover' the curriculum, may explain the empirically weak connection they found between teachers' MKT and how teachers involve students in meaning-making and reasoning. Mediating factors, then, render the use of at least some aspects of teachers' knowledge conditional, thus influencing the role of MKT in practice.

Need for content-specific research on MKT. The details of MKT are content-specific (Hashweh, 2005) and prior research on MKT has lacked a focus on the meanings teachers associate with specific mathematics (Thompson, 2013). We therefore focus our research on a specific topic: exponential functions. We chose exponential functions because it is an important topic in the secondary curriculum (Barker & Ganter, 2004; Common Core Standards Writing Team, 2013). In addition, exponential functions are taught in several high school courses (e.g., Algebra II, Pre-Calculus, and College Algebra) which provides more choice of classrooms for research.

Need for focus on the relationship between teachers' MKT and student learning. Scholars, educators, and preliminary research suggest an association between the nature of teachers' MKT and student achievement (Hill, Rowan & Ball, 2005). This link motivates the need to better understand MKT. Tschoshanov (2011) found that teachers who had a better understanding of mathematical concepts and connections tended to have better student achievement and lesson quality. However, the link between teachers' MKT and student learning merits more study. We intend to examine this link by focusing on how teachers' decisions involving MKT influence students' opportunity to reason mathematically. We draw on the premise that "mathematical reasoning is the foundation for the construction of mathematical knowledge" (Ball, Lewis, & Thames, 2008, p. 41). Therefore, students' opportunity to reason should be associated with student learning and thus a worthwhile indicator of the quality of a teacher's decision making with respect to student learning. This analysis can provide insight into the way in which MKT influences student achievement via teachers' decision making.

1.1 Intellectual Merit

Since the articulation by Shulman (1986) that teachers need knowledge related to the content they teach that is particular to teaching, researchers have been working to define and operationalize this idea. In mathematics education, much of this work has focused on describing components of this knowledge (Ball, Thames, & Phelps, 2008; Davis & Simmt, 2006; Rowland, Huckstep, & Thwaites, 2005), assessing teachers' knowledge (Baumert et al., 2010; Hill et al., 2008), and exploring the relationship between this knowledge and student learning (Baumert, 2010; Tchash

- Creation and dissemination of course materials for the Culture in the Math Classroom (CIMC) course that supports teacher to build their capacity as cultural responsive teachers at the CIMC conference attended by XX mathematics teacher educators

Pathways to Calculus: Disseminating and Scaling a Professional Development Model for Algebra Through Precalculus Teaching and Learning; NSF 1050721; $2.1M; 09/11 – 08/14
Dr. Oehrtman was co-PI on the Pathways to Calculus project.

Intellectual Merit Pathways to Calculus is a Phase II project was designed to research and address the major barriers to teachers incorporating MKT they had previously developed through participation in a targeted Math and Science Partnership. Since existing curriculum severely limited the teachers' ability to act on their MKT, we authored a research-based pre-calculus curriculum developing these central concepts and reasoning skills (Carlson, Oehrtman, & Moore, 2013). Our research contributed to knowledge of implementation of our professional development model at the K-12, community college, and university levels (Madison et al., accepted; Moore & Carlson, 2012; Moore, 2012).

Broader Impacts This research contributed knowledge and tools for scaling up the Pathways professional development model and produced insights about factors that contribute to teacher transformation to support students in developing the capacity and confidence to solve novel problems and construct deeper and more connected understanding of the central ideas of a course.

Collaborative Research: Project CLEAR Calculus: Coherent Labs to Enhance Accessible and Rigorous Calculus Instruction; NSF DUE 1245021; $134,218; 07/13 – 06/16
Dr. Oehrtman is PI on Project CLEAR Calculus.

Intellectual Merit. Project CLEAR Calculus is a research-based effort to make calculus conceptually accessible to more students while simultaneously increasing the coherence, rigor, and applicability of the content students are learning. (Oehrtman, Swinyard, & Martin, 2014a; Oehrtman, Swinyard, & Martin, 2014b; Dibbs & Oehrtman, 2014)

Broader Impacts The project develops quality instruction in introductory calculus sequences, disseminates critical instructor support materials, expands the use of the project labs to successful implementation at other institutions, and assesses student outcomes to characterize the range of variation of prior implementation results and contribute to the broader research knowledge of student learning in calculus.

2. Research Questions, Methods, and Activities

Our research questions are:

1. (Enactment) What are the decisions expert teachers make while engaged in the work of teaching exponential functions that involve MKT?
2. (Nature) What is the MKT these teachers draw on while making these decisions?
3. (Mediation) What factors mediate the use of teachers' MKT in their decision making?
4. (Quality) What is the quality of these teachers' decisions with respect to students' opportunity to reason mathematically and how is this quality related to the types of decisions they make, the nature of their MKT, the factors that mediate the decisions?

2.1 Teacher Recruitment and Selection

We will recruit 12 expert teachers over two years in Algebra II, Pre-Calculus, and College Algebra as these are common high school courses with content related to exponential functions. We draw on Palmer et al. (2005) to identify the characteristics of an expert teacher. The expert teacher is someone who is state licensed in secondary mathematics and has taught at least five years and has taught the course under study at least two years. In addition, we will rely on social recognition (reputation) by at least two constituents (administrators, peers, university faculty). We will assess the relevance of the social recognition based on the degree to which the justification provided for a teacher's expertise aligns with Berliner's (2001) 13 prototypic features of teacher expertise. We will also evaluate the degree to which a teacher's district and/or state assessment data provides evidence of consistent growth in student achievement. Finally, we will ascertain if a teacher has additional experience or credentials that suggest expertise, such as having obtained National Board Certification.

We chose to focus on expert teachers for this study because we want to understand the MKT of teachers with a mature practice who have proved to be successful in the classroom, at least with respect to secondary mathematics teachers in general. Practitioners and scholars have long noted the differences between novice and expert teachers; at this point in our research program, we want to develop an understanding of the MKT of teachers who have passed through the novice stage. Future studies might focus on novice teachers to better understand how to develop their MKT.

Each year, we will select a cohort of 6 teachers, 4 from Colorado and 2 from Oklahoma, as we have researchers in both states (Table 1). We will recruit teachers by drawing on the contacts of the research team and by working with officials in the Colorado and Oklahoma State Departments of Education. We will begin by approaching mathematics curriculum coordinators, mathematics coaches, principals or other administrators from several different districts. Our goal is to have teachers from different districts or from different schools within a district to increase the variety of contexts of the teacher participants. Teachers who agree to participate in the study will receive the appropriate informed consent per IRB requirements. In addition, teachers will receive $125 for each lesson observation (we expect 3-5 observations per teacher) and $250 for two additional interviews.

Table 1. Summary of the 12 research participants

Course	**Cohort 1**		**Cohort 2**	
	CO	OK	CO	OK
Algebra II	2	1	2	1
College Algebra or Pre-Calculus	2	1	2	1

2.2 Data Collection

We will collected teacher and student demographics to provide a picture of the population we are studying. Our main data collection efforts will be around documenting teachers' practice.

Teacher and Student Demographics

Once the selected teachers have agreed to participate in the study, we will collect demographic information on the teacher participants and their students. We will collect the teachers' gender, race/ethnicity, secondary mathematics certification status, highest degree earned, other teaching certifications, number of years teaching, number of years teaching secondary (7-12) mathematics, number of years teaching in their current district, and years teaching the current course. We will collect students' gender, race/ethnicity, special needs identification, English language learner identification, socioeconomic status indicator, age, and whether the student is repeating the course. These data will be used to provide context for our data analysis and findings. Data will be sanitized and secured per IRB requirements.

Data from Teacher Practice

We will gather data on multiple aspects of each teacher's practice related to teaching exponential functions. Based on available district planning documents, we expect each teacher will spend 3-5 days of instruction on exponential functions in a given course.

For each lesson, we will collect data as follows:

- Pre-observation interview (60-90 minutes each)
- Class observation (~60 minutes each)
- Post-observation interview (60-90 minutes each)

Following all of the classroom observations, we will conduct two additional interviews

- Final observation interview (60-90 minutes)
- MKT interview (60-90 minutes)

We have generated detailed protocols for each of the types of interviews and observations that we will conduct with teachers. We have also developed an overarching approach we call our general interview protocol This general protocol serves the purpose of aligning our interview methods with our theoretical perspective and data analysis methods. Our underlying premise is that teachers use MKT to make decisions in their work of teaching. Therefore, we are interested in understanding what decisions teachers are making in their planning, instruction, and assessment and the rationale for those decisions. Decision rationale that relies on any mathematical knowledge or thinking is what we identify as enactment of MKT. Our general interview protocol, then, is based on surfacing teacher decisions and the rationale for those decisions, with a particular goal of determining any mathematical thinking embedded in the decision rationale.

Another key element in our theoretical perspective is that contextual factors mediate teachers' use of their current MKT. For information related to the delivery of the lesson, we may show the teacher video clips from the lesson and ask them to explain their decision-making for the episodes shown in the video.

The second post-observation interview will be the MKT Interview, which will be common to all the teachers. In this interview, we will pose three instructional scenarios to teachers and ask them to consider how they would address the issues in each scenario. The math content in each scenario will be an important idea related to understa

Activity	**Decision**	**Mathematical Knowledge**	**Rationale and Context**
Instruction	Teacher accepts the response “Exponential function because of the word ‘triple.’” to her question of “What type of function is problem 1c?”	The word ‘triple’ in a word problem indicates multiplicative change, indicating the word problem represents an exponential function	Students are working on problems that are known to model either exponential or linear functions
Instruction	Teacher response to the student question “What is x?” is: “We don’t know.”	The exponent x in the equation $y = a{\cdot}b^x$ is a variable and so does not represent a specific number	Student posed the question “What is x?” when the class is working on finding the equation for a word problem that represents an exponential situation (worksheet problem 1c)
Assessment (reflection)	The discussion in class about understanding the exponential growth of dividing by 2 is the same as multiplying by ½ went well.	The growth factor in exponential functions can be thought of as multiplying or dividing by a number. However, division can also be interpreted as multiplication by using the reciprocal of the number being used for division.	Teacher believes it’s important for students to be able to think about all 4 arithmetic operations in terms of addition and multiplication with subtraction and division being inverse operations of addition and division because in her experience mathematics most often Twgwta

Once we have produced descriptions of the opportunities to reason for each teacher, we look for any relationships that exist between teachers' decision making and the nature of the students' opportunities to learn. For this we will draw on our previous analysis of the types of decisions teachers make that involve MKT, the descriptions of teachers' MKT, and the contextual factors that influenced the use of teachers' MKT. Using these data we will generate hypotheses about possible relationships and then use constant comparative techniques to validate, refine, or reject these hypotheses.

While we are not explicitly studying the relationship between teachers' MKT and a measure of student learning, we will be giving a pre- and post-assessment on exponential functions to students. We have been working on developing and validating this assessment for two years. Despite a small sample size (N = 12), we want to perform preliminary quantitative data analysis to i) determine if the hypothesis is reasonable that richer MKT of teachers as measured by the quality of the opportunities to reason is associated with greater mathematical knowledge and skills of their students, and ii) to set a foundation for future research exploring this link. To that end, we propose to categorize teachers based on the richness of their MKT. Our plan is to create two groups of teachers, a high (richer) group and a low (less rich) group. Then we will perform an analysis of covariance with our two categories of teachers as the independent variable, students' post-assessment scores of the exponential functions assessment as the dependent variable, and students' pre-assessment scores of the exponential functions assessment as the covariate. This analysis will test whether there is a statistically significant difference between teachers with richer MKT and teachers with less rich MKT on students' post-assessment scores controlling for pre-assessment scores. Because these results would be preliminary, they are not generalizable but rather would be used as a basis for further study.

3. Project Management Plan

In this section, we outline the roles and responsibilities of members of the INFORMS MKT research

3.2 Project Implementation Schedule

Table 3 contains a timeline of the research activities which we detailed in the Data Collection and Analysis section and the evaluation activities which we detail in the Project Evaluation section. Before the project begins, we will submit an IRB for institutional approval and validate the exponential functions assessment. The major project activities during the first two years are collecting and analyzing data for two cohorts of teachers. Towards the end of year two, we begin cross-cohort analysis which continues in

Time Frame	Activity Description
Nov – April	Present preliminary findings

enough back up teachers, we will first look for expert teachers in our current districts. If no appropriate teacher is available, we will begin working with another district to identify teacher participants.

We are providing a financial incentive (up to $875 per teacher) to compensate teachers for their time outside of contract time needed for the interviews. The financial incentive is likely to reduce attrition. If the financial incentive is viewed as too low at the district level or by individual teachers, we can increase the financial incentive appropriately for all teachers.

4. Project Evaluation

We seek external, objective evaluation to provide ongoing, critical review of our research design and activities, including our theoretical framework, data collection protocols, analyses, and reporting (Sutton & Callow-Heusser, 2014) to ensure that we are engaging in high quality research that is appropriately informed by the literature and informing the research community (Guthrie et al., 2013). Two external evaluators with expertise in the field have already agreed to serve: Dr. Mark Thames (University of Michigan) and Dr. Amy Ellis (University of Wisconsin-Madison). We will engage other external

Timeline	External Evaluation Task	Data for Task
Aug 2017	Review documentation of our research methods	Data collection protocols, data analysis protocols, updated report of the evolution of our research methods
Aug – Sept 2017		