T. A. Engstrom

II. MINIMAL COUPLING FORMALISM

Consider a quantum particle with wave function $\Psi(\;, t) = \Psi_1(\;, t) + i\Psi_2(\;, t)$, Ψ_1 and Ψ_2 being real, that satisfies the time-dependent Schrödinger equation $i\hbar\, \partial_t \Psi = \hat{H}\Psi$. This

Case 1: Spatial derivatives of P are negligible, and ML/B and PL^2/B are both $\ll 1$ so that we may retain terms only to linear order in these quantities in the nondimensionalized equations of motion (lengths measured in units of L, time in units of $L^2\sqrt{\ /B}$). On the quantum side, this case corresponds to the semiclassical approximation with weak potentials. On the elastic side, it corresponds to small terminal twist angle $\ = (1+\nu)ML/B$ where ν is Poisson's ratio, and P well below the Euler buckling threshold $\ ^2B/L^2$. Equations (6) reduce to

$$-\ \ddot{\Psi}_1 = B\Psi_1'''' + M\Psi_2''' + P\Psi_1'', \tag{7a}$$

$$-\ \ddot{\Psi}_2 = B\Psi_2'''' - M\Psi_1''' + P\Psi_2''. \tag{7b}$$

To linear order in ML/B, the twist-induced tension contribution to P is not accounted for, and all elastic parameters become independent of one another. Incidentally, *static* heli-

not in a thermal equilibrium sense since here the total energy (found by changing the signs of the last three terms in Eq. (12) and integrating over x) is not also periodic in .

Letting Eqs. (14) describe the periodic straight rod and setting $M = P = 0$ yields a set of de Broglie relations: $p_x = \int dx\, g = \hbar k_n$ (using one of the first two methods for g) and $E = \hbar\omega_n$ (equal parts kinetic and potential energy and also apmore exotic physics, such as the Zak phase [32], but in a classical setting.

ACKNOWLEDGMENTS

I thank T. Allen and J. Chaloupka for useful comments on an early version of this paper and D. Rowland for an enlightening discussion about momentum density of strings and rods.

APPENDIX A: ENERGY-MOMENTUM TENSOR

The energy-momentum tensor, also known as the stress-energy tensor, is usually derived assuming the Lagrangian density depends on field variable derivatives only up to first order. Here, we derive it (in the notation of Ref. [33]) for a Lagrangian density, such as Eq. (12), that depends on field variable $u(x)$ derivatives up to second order: $\mathcal{L} = \mathcal{L}(u, \nabla u, \nabla\nabla u, x)$. The Euler-Lagrange equations are

$$\frac{\partial \mathcal{L}}{\partial u_i} - \partial_\alpha \frac{\partial \mathcal{L}}{\partial u_{i,\alpha}} + \partial_{\alpha\beta} \frac{\partial \mathcal{L}}{\partial u_{i,\alpha\beta}} = 0. \tag{A1}$$

Under an ϵ-family of transformations of the field variables, and writing $\delta \equiv d/d\epsilon$ at $\epsilon = 0$,

$$\delta\mathcal{L} = \frac{\partial \mathcal{L}}{\partial u_i}\delta u_i + \frac{\partial \mathcal{L}}{\partial u_{i,\alpha}}\delta u_{i,\alpha} + \frac{\partial \mathcal{L}}{\partial u_{i,\alpha\beta}}\delta u_{i,\alpha\beta}. \tag{A2}$$

Combining the previous two equations yields

$$\delta\mathcal{L} = \partial_\alpha\left[\left\{\frac{\partial \mathcal{L}}{\partial u_{i,\alpha}} - \partial_\beta\left(\frac{\partial \mathcal{L}}{\partial u_{i,\alpha\beta}}\right)\right\}\delta u_i + \frac{\partial \mathcal{L}}{\partial u_{i,\alpha\beta}}\right.$$